

Section 6.4 - Equations Involving Inverse Trig Functions

Definition: $\alpha = \arcsin \beta \Leftrightarrow \sin \alpha = \beta$
or

If $\sin x = y$ then $\sin^{-1} y = x$ or $\arcsin y = x$

Example: rewrite $\sin 45^\circ = \frac{\sqrt{2}}{2}$ as an INVERSE

Examples: Solve for x: (in terms of y)

1. $y = 3 \cos 2x$

$$\begin{aligned} & \cancel{y = 3 \cos 2x} \\ & \cancel{y} = \cancel{3} \cos 2x \\ & \cancel{y} = \cancel{2} \cos x \\ & \cancel{y} = \cancel{2} \cos x \\ & \cancel{y} = \cancel{2} \cos x \end{aligned}$$

$$\cancel{y} = \cancel{3} \cos 2x$$

$$\frac{y}{3} = \cos 2x$$

$$\cos^{-1}(\frac{y}{3}) = 2x$$

$$\frac{\cos^{-1}(\frac{y}{3})}{2} = x$$

2. $y = \arctan(2x - 1) + 5$

$$y - 5 = \arctan(2x - 1)$$

$$\tan(y - 5) = 2x - 1$$

$$\frac{\tan(y - 5) + 1}{2} = x$$

Examples: Solve for x: EXACT values

1. $2 \arcsin x = \pi$

$$\sin^{-1} x = \frac{\pi}{2}$$

$$\sin \frac{\pi}{2} = x$$

$$1 = x$$

2. $\arccos x = \arcsin \frac{1}{5}$

$$x = \cos(\arcsin \frac{1}{5})$$

$$x = \cos\left(\frac{\sqrt{1 - (\frac{1}{5})^2}}{2\sqrt{6}}\right)$$

$$x = \frac{2\sqrt{6}}{5}$$

2. $\sin^{-1} x + \tan^{-1} \sqrt{3} = \frac{2\pi}{3}$

$$\sin^{-1} x = \frac{2\pi}{3} - \tan^{-1} \sqrt{3}$$

$$\sin^{-1} x = \frac{2\pi}{3} - \frac{\pi}{3}$$

$$\begin{aligned} \sin^{-1} x &= \cancel{\frac{2\pi}{3}} - \cancel{\frac{\pi}{3}} \\ \sin \cancel{\frac{\pi}{3}} &= x \\ x &= \frac{\sqrt{3}}{2} \end{aligned}$$