Section 6.3 - Trigonometric Equations II (Multiple & Half Angles)

Solve: $0° \leq x < 360°$

1. $2\cos x = 1$ (same as 6.2)

$\cos x = \frac{1}{2}$

$\theta' = 60°$ QI, IV

Answers: $\theta = 60°$ ($\pi/3$) and $300°$ ($5\pi/3$)

We need to know what

$x = ?$

$3x=60$
$3x=300$
$3x=420$
⋮

$\therefore 3x = 60, 300, 420, 660, 780, 1020$

and $x = 20°, 100°, 140°, 220°, 260°, 340°$

Answers!!   all b/t $0° \leq x < 360°$

2. $2\cos 3x = 1$
$\underline{\underline{\text{different}}}$

Step 1: solve for trig fxn
$\cos 3x = \frac{1}{2}$

Step 2: Find Quadrants + # of times around the ☉
$0° \leq x < 360° \Rightarrow$ we need $3x$ ($\overset{or}{3 ☉}$)
$\therefore 0° \leq 3x \leq 1080°$ (makes 3 circles ☉)

Step 3: $\frac{1}{2}$ is positive $\therefore$ QI and QIV
Find $\theta' = 60°$

Step 4 Find all answers!!

$3x = \begin{cases} \text{1st ☉} & 60° & 300° \\ \text{2nd☉} & 60°+360°=420° & 300°+360° = 660° \\ \text{3rd☉} & 420°+360°=780° & 660°+360° = 1020° \end{cases}$

3. $2\sin\frac{x}{2} = 1$

$\sin\frac{x}{2} = \frac{1}{2}$

Sine is (+) in QI, II

$0° \leq x < 360°$

$0° \leq \frac{x}{2} < 180°$ (½ circle)

$\theta' = 30°$ QI = 30°   QII = 150°

$\frac{x}{2}$	30°	150°
x	$30(2)=60°$	$150(2)=300°$

Answers: $60°, 300°$

4. $4\sin x \cos x = \sqrt{2}$

Find identity 1st + rewrite!!

$2\underbrace{(2\sin x \cos x)}_{\text{identity}} = \sqrt{2}$

$2(\sin 2x) = \sqrt{2}$

$\underline{2 ☉'s}$ $\sin 2x = \frac{\sqrt{2}}{2}$ $\theta' = 45°$

First ☉	45°	135°
2nd ☉ (+360°)	405°	495°

$2x = 45, 135, 405, 495$

$2x=45$
$2x=135$
$2x=405$
$2x=495$

Ans: $x = 22.5°, 67.5°, 202.5°, 247.5°$

Rules:
1. **Set equal to zero and factor**
   (Do Not Divide a Function Out)
2. **Only one size angle per factor.**
3. **Only one trig function per factor.**
4. **Multiple angles result in multiple sets of answers**

Solve $0° \leq x < 360°$

1. $\cos 2x = \cos x$

   ⌐ Find identity

   $\underset{\cos 2x}{\underline{2\cos^2 x - 1}} = \cos x$

   $2\cos^2 x - \cos x - 1 = 0$

   $(2\cos x + 1)(\cos x - 1) = 0$

   $2\cos x + 1 = 0 \longrightarrow \cos x = -\frac{1}{2}$ ref $\angle$ 60°

   $\cos x = 1$ $\boxed{x = 0°}$ Q: II, III $\boxed{x = 240°}$

   $\boxed{x = 120°}$

   omit #33

2. $2\cos 2x \sin 3x = \sqrt{2} \cos 2x$

   $2\cos 2x \sin 3x - \sqrt{2}\cos 2x = 0$

   $\cos 2x (2\sin 3x - \sqrt{2}) = 0$

   $\cos 2x = 0$ or $2\sin 3x - \sqrt{2} = 0$ $0 \leq x < 360$

   $\theta = 2x$ $x = 90, 270_{+360}$ $2\sin 3x = \sqrt{2}$ $0 \leq 3x < 1080$

               $450, 630$

   $0 \leq x < 360$ $2x = 90$ $\boxed{x = 45}$ $\sin 3x = \frac{\sqrt{2}}{2}$ Q: I, II **45°, 135°**

   $x = 90, 2$ $0 \leq x < 720$ $2x = 270$ $\boxed{x = 135}$ $\frac{\sqrt{2}}{2}$ $3x = 405, 495°$

            $2x = 450$ $2x = 630$ $x = 15°, 45°, 135°, 165°,$ $765°, 855°$

              $\boxed{x = 225}$ $\boxed{x = 315}$ $255°, 285°$